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TURBULENT HEAT AND MASS TRANSFER FROM A WALL WITH PARALLEL ROUGHNESS RIDGES

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Abstract—As it is known, the general equation for the coefficient of heat (or mass) transfer between a rough wall and a turbulent fluid flow can be derived with the aid of general dimensional and similarity considerations supplemented with some additional physical arguments. The equation is specified here for the case of a wall covered with two-dimensional roughness in the form of widely spaced parallel ridges perpendicular to the stream direction. The constant coefficients of the derived equation are approximately estimated from the available data on mean temperature or concentration profiles in wall turbulent flows over two-dimensional roughness. The final results of the calculations agree satisfactorily with all of the experimental data on turbulent heat and mass transfer in pipes and channels with regularly repeated parallel roughness ridges and in boundary layers on plates with two-dimensional roughness of the same form.

NOMENCLATURE

a,	constant in the equation for the boundary-layer thickness L;
a_H, a'_H, a''_H,	dimensionless coefficients in the equation for ε_H in the neighbourhood of the wall;
A,	constant in the universal logarithmic velocity profile equation;
b_1, b_2,	constants in the equation for the function $\beta(Pr)$ in a smooth-wall flow;
b'_1, b'_2,	constants in the equation for the function $\beta(Pr, h_+)$ in a flow along a wall with closely spaced roughness elements;
b''_1, b''_2,	constants in the equation for the function $\beta(Pr, h_+)$ in a flow along a wall with two-dimensional roughness;
B^{-1},	Owen and Thomson's parameter describing temperature or concentration variation within the roughness sublayer;
B',	constant in the velocity profile equation;
c_f,	skin friction coefficient;
c_h, C_h,	dimensionless heat- or mass-transfer coefficients (i.e. thermal or diffusion Stanton numbers) based on maximum and bulk values, respectively;
c_p,	specific heat capacity at constant pressure (to be replaced by unity in case of mass transfer);
C, C',	constant in equations for $\beta(Pr, h_+)$;
D,	pipe diameter;
h,	mean height of roughness elements;
h_0,	roughness parameter;
h_s,	height of the equivalent sand roughness;
$h_+^{(0)}$,	"threshold value" of h_+;
j,	heat or mass flux;
k,	exponent in equation (7) for $\beta(Pr, h_+)$;
L,	typical vertical size of the flow;
Nu, Nu_x,	$C_h RePr$ and $c_h RePr$, Nusselt numbers;
p,	distance between two-dimensional roughness ridges;
Pe,	$RePr$, Péclet number;
Pr,	thermal or diffusion Prandtl number;
Re,	$U_1 L/\nu$ or $U_b D/\nu$, Reynolds number;
Re_x,	$U_1 x/\nu$;
Re_*,	$h_0 u_*/\nu$;
u_*,	$(\tau_w/\rho)^{1/2}$, friction velocity;
U,	mean velocity;
x,	distance from leading edge of a plate in a boundary layer flow;
y,	coordinate measured normal to a wall.

Greek symbols

α,	coefficient at the logarithmic term in equations (3);
β, β_1,	constant terms in equations (3);
$\tilde{\beta}$,	$\beta + \alpha \ln h_+$;
γ,	exponent in the equation for the function $\varepsilon_H(y)$ in the neighbourhood of a wall with two-dimensional roughness;
δ_3,	energy-loss thickness of a boundary layer;
δ_v,	viscous sublayer thickness;
Δ_2,	enthalpy thickness;
Δ^{-1},	$(\theta_w - \theta_1)/(\theta_w - \theta_b)$, correction factor for transition from c_h to C_h;
$\varepsilon_H, \varepsilon_M$,	eddy (thermal or mass) diffusivity and eddy viscosity;
η,	dimensionless distance from a wall for outer region of a flow;
η_1,	h/L, dimensionless roughness height;

θ,	mean temperature or mean concentration;
θ_*,	$j_w/c_p\rho u_*$, heat flux temperature (or mass flux concentration);
ν,	kinematic viscosity;
ρ,	density;
$\sigma_1, \sigma_2, \ldots$,	dimensionless parameters describing shape and area distribution of roughness elements;
τ,	shear stress;
φ, φ_1,	universal functions;
χ,	molecular diffusivity for heat or mass transfer.

Subscripts

b,	bulk quantities;
c,	calculated quantities;
m,	measured quantities;
r,	quantities for a completely rough wall;
s,	quantities for a smooth wall;
w,	wall quantities;
$+$,	dimensionless quantities;
1,	maximum or minimum quantities at $y = L$.

INTRODUCTION

THE FIRST investigations of heat and mass transfer between a rough wall and turbulent fluid flow were made more than fifty years ago. During all these years plenty of valuable experimental and theoretical works have been devoted to the problem. The references to many of them can be found in recent monograph [1] and in other works cited in the present paper. Nevertheless it is impossible to say at present that the problem of a turbulent heat and mass transfer at rough walls is completely solved.

The present paper is devoted to derivation of correlations for turbulent heat and mass transfer at a wall covered with two-dimensional roughness. Namely, we consider roughness in the form of regularly repeated parallel ridge-like protrusions perpendicular to the stream direction. The analysis is developed in the frames of the general approach to the study of heat and mass transfer in wall flows at high enough Reynolds and Péclet numbers proposed by Fortier [2, 3] and the present authors [4–6]. The approach is essentially similar to Millikan's [7] derivation of a skin-friction law for smooth- and rough-wall turbulent pipe and channel flows. It is based primarily on general dimensional and similarity arguments having a clear physical meaning.

Let us assume that the rough wall is uniform, while the turbulent flow is steady and parallel (i.e. all mean fluid dynamic values depend only on the distance y from the wall, but not on the time t and coordinates x and z). The wall temperature (or wall concentration of the transported substance) θ_w is assumed to be constant and different from the temperature (or concentration) of the fluid. It is also assumed that $\nu/\chi = Pr \gtrsim 1$, i.e. we shall not consider heat transfer in rough-wall flows of liquid metal (since at present there are no reliable experimental data on such heat transfer).

Dimensional considerations imply that in a wall layer of the flow (i.e. at $h < y \ll L$, where h is a protrusion height and L is a typical vertical size of the flow, e.g. pipe radius, channel halfwidth, or boundary-layer thickness), the mean temperature profile† will satisfy the temperature wall law of the form

$$\theta_w - \theta(y) = \theta_* \varphi(y_+, Pr, h_+, \sigma_1, \sigma_2, \ldots) \tag{1}$$

where $\theta_* = j_w/c_p\rho u_*$ is the so-called heat-flux temperature, $y_+ = yu_*/\nu$, $h_+ = hu_*/\nu$ and $\sigma_1, \sigma_2, \ldots$, are dimensionless parameters characterizing the shape of roughness elements, their distribution over the wall surface and (in cases when not all of them are identical) the scatter of their dimensions and shapes. On the other hand, at $y \gg \nu/u_*$ and $y \gg h$, i.e. in the core of the tube or channel flow or in the outer part of the boundary-layer flow, the temperature defect law is valid, if the numbers $Re = U_1 L/\nu$ and $Pe = U_1 L/\chi = RePr$ are high enough. This law has the form

$$\theta(y) - \theta_1 = \theta_* \varphi_1(\eta) \tag{2}$$

where $\theta_1 = \theta(L)$ and $\eta = y/L$. Let us also assume that the Reynolds and Péclet numbers are so high that there is an overlap layer in which both the laws (1) and (2) apply simultaneously. Then the comparison of the two laws implies that both functions $\varphi(y_+)$ and $\varphi_1(\eta)$ must be logarithmic in the overlap layer:

$$\varphi(y_+, Pr, h_+, \sigma_1, \sigma_2, \ldots) = \alpha \ln y_+ + \beta(Pr, h_+, \sigma_1, \sigma_2, \ldots),$$
$$\varphi_1(\eta) = -\alpha \ln \eta + \beta_1 \tag{3}$$

(cf. Monin and Yaglom [8], Sections 5.5 and 5.7). If we now substitute (3) into (1) and (2) and then add the results, we shall obtain an expression for $\theta_w - \theta_1$ leading to the heat (or mass) transfer law of the form

$$c_h = \frac{(c_f/2)^{1/2}}{\alpha \ln[Re(c_f/2)^{1/2}] + \beta(Pr, h_+, \sigma_1, \sigma_2, \ldots) + \beta_1} \tag{4}$$

where $c_h = j_w/c_p\rho U_1(\theta_w - \theta_1)$ is the heat- (or mass-) transfer coefficient (Stanton number) and $c_f = 2(u_*/U_1)^2$ is the skin-friction coefficient. When the values of Re and c_f are given, the corresponding c_h can be evaluated with the aid of equation (4), if the values of the coefficients α, β and β_1 are known.

The numerical coefficients α and β_1 do not depend on the wall parameters, i.e. they are the same for all wall flows of the same type. The available data were discussed in [5, 6] where the following recommendations were given: $\alpha \approx 2.12$ in all the cases, $\beta_1 \approx 0.5$ for circular pipe and plane channel flows and $\beta_1 \approx 2.35$ for boundary-layer flows. The same values α and β_1 are used in the present paper. Thus for the possibility

† We shall henceforth talk mostly about heat transfer and temperature field $\theta(y)$. However, all subsequent arguments can be applied to mass transfer if the meaning of θ, j_w and χ is changed accordingly and it is assumed that $c_p = 1$.

The values of $\theta = \theta(y)$ in a wall-roughness sublayer of the thickness of the order of h are meant as area-mean values (i.e. mean values over the plane y = const). Similar meaning has the value j_w of the wall flux.

to use equation (4) the value of $\beta = \beta(Pr, h_+, \sigma_1, \sigma_2, \ldots)$ must be estimated. This problem is more complicated than the estimation of α and β_1 and it will be considered in detail in the following section.

2. HEAT- AND MASS-TRANSFER LAW FOR TURBULENT FLOWS ALONG A WALL COVERED WITH TWO-DIMENSIONAL ROUGHNESS

In the paper [5] devoted to heat and mass transfer in smooth-wall turbulent flows the equation

$$\beta(Pr) = 12.5Pr^{2/3} + 2.12 \ln Pr - 5.3$$

was recommended for $\beta = \beta(Pr)$ in a case when $h_+ = 0$. This equation was derived by a simplified analysis of the behaviour of $\theta(y)$ within the viscous sublayer and by the treatment of all the available data on mean temperature profiles in smooth-wall turbulent flows. Moreover the term $2.12 \ln Pr$ was included only to fit the data for liquid metal flows ($Pr \ll 1$). Therefore in [6] a simpler smooth-wall equation for $\beta(Pr)$ was recommended at $Pr \gtrsim 1$ (i.e. for all the cases with the exception of heat transfer in rough-wall flows of liquid metals), namely

$$\beta(Pr) = \beta_s = 12.5Pr^{2/3} - 6. \tag{5}$$

Equation (5) fits all the smooth-wall data at $Pr \geqslant 0.6$ with the same accuracy as the equation for $\beta(Pr)$ recommended in [5]. However, no smooth-wall equation for β can be applied to a rough-wall case that requires a special study.

Numerous data on turbulent heat and mass transfer at a wall with closely spaced three-dimensional roughness are analysed in [6]. It has been found that all of the data treated in [6] can be described with a satisfactory accuracy (which is somewhat less than that achieved in [5] for heat and mass transfer at smooth walls) by equation (4) where $\beta = \beta(Pr, h_+)$ is given by the single equation for all the considered types of roughness. In other words the results of [6] imply that the dependence of β on the parameters $\sigma_1, \sigma_2, \ldots,$ describing specific features of the wall geometry, turns to be so weak, that it can be neglected in the first approximation when only the walls covered with closely spaced roughness elements are considered. For dynamically completely rough walls (i.e. at sufficiently large h_+) the following equation is recommended in [6]:

$$\beta(Pr, h_+) = \beta_r = 0.55h_+^{1/2}(Pr^{2/3} - 0.2) - 2.12 \ln h_+ + 9.5. \tag{6}$$

The general form of the equation was found by the consideration similar to those used in [5]. For transitional flows along dynamically slightly rough walls the linear interpolation between $\beta_r(Pr, h_+)$ and $\beta_s(Pr)$ is suggested in [6].

The applicability of the same function $\beta(Pr, h_+)$ independent of any additional parameters $\sigma_1, \sigma_2, \ldots,$ to many types of roughness implies that the single heat-transfer law can be used in the first approximation for a great variety of different rough walls. It is clear, however, that the law cannot be quite universal, i.e. it cannot be applicable to all existing types of roughness. In particular it has already been pointed out in [6] that some of the regular deviations of the experimental points from the theoretical curves in the figures of this paper may be explained in some cases by the inaccuracy of the equation for β as applied to specific types of roughness. Such a situation is quite probable, for example, in the case of Nunner's experiments on heat transfer in pipes with the walls roughened by relatively sparse circumferential rings (heat-transfer data for such pipes showed especially poor agreement with theoretical equations recommended in [6]). It has also been pointed out in [6] that Webb *et al.*'s [10] data on heat transfer in pipes with repeated-rib roughness noticeably differ from the results following from (6). Let us also mention recent results of Garratt and Hicks [11] who treated numerous data on heat ($Pr = 0.71$) and moisture ($Pr = 0.62$) transfer into the air from a large number of artificial and natural rough surfaces. They plotted a summary graph of the special dimensionless parameter B^{-1} (expressed through β and roughness parameter h_0) vs dimensionless combination $Re_* = h_0 u_*/\nu$ simply related to $h_+ = hu_*/\nu$ (the dependence of B^{-1} on Pr was not considered in [11] since the data were rather crude and two values of Pr used differed but slightly). According to [11] the parameter B^{-1} (and hence β) by no means can be presented at fixed Pr as a single-valued function of Re_* (i.e. of h_+). Namely, at large Re_* the values of B^{-1} for surfaces with two-dimensional roughness (i.e. regularly repeated parallel rows of protrusions) essentially differ from B^{-1} in case of more irregular and dense three-dimensional roughness. All this shows, that in cases of "two-dimensional roughness" we cannot use equation (6) that was suggested in [6] for a wall with closely spaced roughness elements. Hence the problem of the determination of β for a wall with two-dimensional roughness requires a special study.

It is natural to expect that the sparse two-dimensional roughness in the form of repeated parallel ridges leads to weaker deterioration of turbulent transfer in the gaps between protrusions, than closely spaced three-dimensional roughness with the same value of h_+. It is suggested in [6] that the thermal eddy diffusivity $\varepsilon_H(y)$ in the gaps between closely spaced three-dimensional protrusions can be described by the equation $\varepsilon_H = a'_H \nu h_+^{-3/2} y_+^3$ where a'_H does not depend on h_+ and y_+. If this suggestion is correct, then it is reasonable to assume that in cases of two-dimensional roughness whose ridges do not contact with one another (but let us say, follow each other at a prescribed distance p), the thermal eddy diffusivity $\varepsilon_H(y)$ near the wall in the gaps between the ridges, averaged over the whole area of these gaps, may be described by the equation $\varepsilon_H(y) = a''_H \nu h_+^{-\gamma} y_+^3$ where $0 < \gamma \leqslant 3/2$. (Let us remind that the equation of the form $\varepsilon_H(y) = a_H \nu y_+^3$ with $\gamma = 0$ describes the smooth-wall case where there are no protrusions at all.) It can easily be seen that if $\varepsilon_M(y) \sim \varepsilon_H(y) \sim h_+^{\gamma} y_+^3$ then the thickness δ_v of the viscous sublayer is given by the equation $\delta_v \sim (\nu/u_*)h_+^{\gamma/3}$ where δ_v is determined by the usual condition that $Re_v = \delta_v U(\delta_v)/\nu$ is of the order of unity and, clearly,

$0 < \gamma/3 \leqslant 1/2$. By repeating all the arguments of [6] (and, in particular, by assuming again that above the level $y = \delta_v$ the variation of the mean temperature is approximately proportional to the variation of the mean velocity), but by using the new values of the eddy diffusivity $\varepsilon_H(y)$ and of the thickness δ_v, we arrive at the relation of the form

$$\beta(Pr, h_+) = f_1'' h_+^k (Pr^{2/3} + b_2'') - \alpha \ln h_+ + C' \quad (7)$$

where $b_1'' > 0$, and $k = \gamma/3$ (i.e. $0 < k \leqslant 1/2$). If we now substitute this result into (4) we find that

$$c_h = \frac{(c_f/2)^{1/2}}{\alpha \ln L/h + b_1'' h_+^k (Pr^{2/3} + b_2'') + C' + \beta_1}. \quad (8)$$

This equation for c_h will be compared later with the existing experimental data on heat and mass transfer at completely rough walls covered with two-dimensional roughness.

The numerical parameters k, b_1'', b_2'' and C' entering into equations (7) and (8) theoretically may depend upon the shape and distribution of roughness ridges, i.e. on the parameters $\sigma_1, \sigma_2, \ldots$. In particular, it seems natural to expect that the exponent k would depend on the ratio p/h describing the density of roughness ridges and would decrease with increase of p/h. The results of the next section show, however, that for many different types of two-dimensional roughness (characterized by markedly different values of p/h) a satisfactory agreement with available experimental data may be achieved when the single collection of values of k, b_1'', b_2'' and C' is used. This circumstance (which, of course, may become incorrect, if the class of the considered rough surface is expanded, the accuracy of the experimental results is increased or stronger requirements to the agreement of experiments with a theory are used) considerably simplifies the practical application of the recommended heat- and mass-transfer law.

3. COMPARISON WITH EXPERIMENTS

The comparison of equation (8) with available data on turbulent heat and mass transfer at walls with two-dimensional roughness is possible only when the numerical parameters k, b_1'', b_2'' and C' are determined. The most simple, though rather crude and indirect, method of determining the values of the above parameters is based on the treatment of experimental data on the coefficient c_h. The data at very high Pr numbers are essentially valuable in this respect, because at $Pr \gg 1$ the term proportional to $Pr^{2/3}$ is the most important in the denominator of the RHS of equation (8). The formula for c_h can therefore be approximately rewritten in a more simple form

$$c_h \approx (b_1'')^{-1} h_+^{-k} Pr^{-2/3} (c_f/2)^{1/2} \quad (9)$$

with only two unknown parameters k and b_1''. Recently, Dawson and Trass [14] have measured electrochemical mass transfer between a metal solid wall with two-dimensional roughness and turbulent flow of an electrolyte flowing along the wall. They have obtained in these experiments the values of c_h for Pr between 390 and 4590. According to the data of Dawson and Trass the single value $k = 1/4$ can be used as the first approximation to k for all types of roughness considered in [14]. (The ratio p/h varied in the measurements by Dawson and Trass in the interval $3.6 \leqslant p/h \leqslant 7.5$, but it will be seen later that the same approximate value of k has, in fact, still wider application.) Moreover, the same data allow approximate estimation of b_1'' and the value of this coefficient also turns out to be rather insensitive to the replacement of one rough surface by another. Finally, Dawson and Trass's data confirm excellently the correctness of the exponent at Pr in equations (7) and (8). If in these equations $Pr^{2/3}$ is replaced by $Pr^{3/4}$ (such a replacement corresponds to the assumption that $\varepsilon_H(y) \sim y^4$, rather than $\varepsilon_H(y) \sim y^3$, near a solid wall), then the data of [14] fail to correlate (8) whatever the values of k, b_1'', b_2'' and C' are.

When the values of k and b_1'' are estimated based on heat- and mass-transfer data at $Pr \gg 1$, the parameters b_2'' and C' may be estimated approximately by the measurements of heat transfer through air (at $Pr = 0.71$). However, such an approach will not be used here, and even the values of k and b_1'' will repeatedly be estimated below in another way. The case is that the knowledge of k, b_1'', b_2'' and C' is, in fact, necessary only to find the equation for the coefficient c_h. Therefore it is very alluring to determine these values independently from any data directly related to heat and mass transfer and then to use the experimental values of c_h to control the parameters estimated above. The most direct method of determining the above parameters is based on the comparison of equation (7) with experimental values of β derived from the measurements of the mean temperature and concentration distributions in rough-wall turbulent flows along a wall covered with two-dimensional roughness (and simultaneous measurements of turbulent fluxes allowing determination of θ_*). Unfortunately the reliable experimental data of such a type remain very scanty and incomplete up to now. Among the known data only some of Chamberlain's measurements [15] proved to be suitable for approximate determination of the coefficient β in the flow along a wall with two-dimensional roughness. In [15] the concentrations of water vapour ($Pr = 0.62$) and ThB radioactive vapour ($Pr = 2.77$) were measured in a number of turbulent air flows over rough walls of different kinds. Data of this paper include the values of u_* and j_w and of the dimensionless velocity and dimensionless concentration $[\theta_w - \theta(y)]/\theta_*$ of water and ThB vapours at the point within the logarithmic layer (at the height $y = 5$ cm over the wall). It is clear that these data allow easy determination of the experimental values $\tilde{\beta}_m$ of the constant

$$\tilde{\beta} = \beta + \alpha \ln h_+ = [\theta_w - \theta(y)]/\theta_* - \alpha \ln(y/h) \quad (10)$$

(cf. [6]).

Only those data of [15] are used in the present paper that are related to two-dimensional roughness (consisting of parallel cylinders, half-cylinders or wavelike ridges placed on a plate with p/h in the range

$2 \leqslant p/h \leqslant 10$). Additionally, the mass-transfer data of Owen and Thomson [13] for two-dimensional roughness are also used for the indirect estimation of $\tilde{\beta}$. In [13] the vertical transport of camphor through an air boundary layer is studied ($Pr = 3.2$). The boundary layer is formed along two rough plates (the roughness of one of them being "two-dimensional") sprinkled with camphor solution. There are no direct data on camphor concentration profiles in [13] but the authors suggest an indirect method of estimation of the dimensionless vertical concentration difference B^{-1} within the roughness sublayer. The obtained estimates of B^{-1} within the roughness sublayer. The obtained estimates of B^{-1} imply approximate evaluation of $\tilde{\beta}$ (cf. [6]). As it is seen from Fig. 1 all the estimates of $\tilde{\beta}$ obtained from the data of [13] and [15] for two-dimensional roughness are described with sufficient accuracy by the equation

$$\tilde{\beta} = \beta + \alpha \ln h_+ = 3.2h_+^{1/4}(Pr^{2/3}+0.3)+3.5. \quad (11)$$

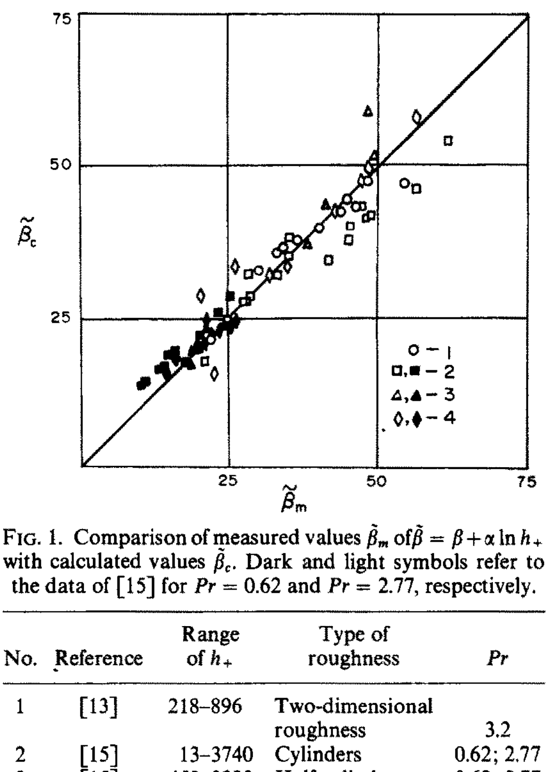

FIG. 1. Comparison of measured values $\tilde{\beta}_m$ of $\tilde{\beta} = \beta + \alpha \ln h_+$ with calculated values $\tilde{\beta}_c$. Dark and light symbols refer to the data of [15] for $Pr = 0.62$ and $Pr = 2.77$, respectively.

No.	Reference	Range of h_+	Type of roughness	Pr
1	[13]	218–896	Two-dimensional roughness	3.2
2	[15]	13–3740	Cylinders	0.62; 2.77
3	[15]	450–3390	Half-cylinders	0.62; 2.77
4	[15]	51–3230	Wavelike ridges	0.62; 2.77

Equation (11) accurately fits general equation (7) and shows that $k = 1/4$, $b_1'' = 3.2$, $b_2'' = 0.3$ and $C' = 3.5$. Since at present there are no more reliable estimates, these values of k, b_1'', b_2'' and C' are used throughout the present paper. Let us also remind that it has already been noted in [6] that the value of C' should apparently be relatively close to the value of the constant B' in the logarithmic velocity profile equation for a rough-wall flow: $U(y)/u_* = A\ln(y/h)+B'$. The above value 3.5 of C' is substantially lower than the value $C' = 9.5$ suggested in [6] for a wall covered with closely spaced three-dimensional roughness. This agrees with the fact that according to Nikuradse $B' = 8.5$ in the logarithmic velocity profile equation for a wall covered with closely spaced three-dimensional homogeneous sand roughness (cf. [8], Section 5.4) while for a wall with two-dimensional roughness the same constant B' takes lower values in the range $3 \leqslant B' \leqslant 8$ (see [10, 14]).

Let us now compare equation (8) with the values of k, b_1'', b_2'' and C' chosen according to (11) with the available data on heat and mass transfer at the wall covered with two-dimensional roughness. When the heat transfer in a boundary-layer flow on a flat plate is studied, it is reasonable to replace the boundary-layer thickness L in equation (8) by more easily measured distance x along the plate from the point of boundary-layer turbulization to the considered cross section of the layer. (In case of substantial roughness x may often be rather precisely identified with the distance from the leading edge of the plate.) Using the known relationship $L = a(c_f/2)^{1/2}x$ where $a = \text{const}$ (see, e.g. [8], Section 5.6) which holds for smooth and for rough plates, and assuming (in accordance with the smooth-wall data) that $a = 0.3$, we obtain, by substitution of the above values of α, β_1, k, b_1'', b_2'' and C' in equation (8) and replacement of L by x, the following form of a heat-transfer law:

$$c_h = \frac{(c_f/2)^{1/2}}{2.12\ln\left(\frac{x}{h}(c_f/2)^{1/2}\right)+3.2h_+^{1/4}(Pr^{2/3}+0.3)+3.5} \quad (12)$$

where $h_+ = (hU_1/\nu)(c_f/2)^{1/2}$. For heat transfer into the air ($Pr = 0.71$) this law assumes particularly simple form:

$$Nu_x = c_h Re_x Pr \approx \frac{Re_x(c_f/2)^{1/2}}{3\ln\left(\frac{x}{h}(c_f/2)^{1/2}\right)+5[(h_+)^{1/4}+1]} \quad (13)$$

Numerous experimental data on heat transfer between a hot plate covered with two-dimensional roughness and air flow along the plate may be found in [16, 17]. These data cover a wide range of shapes, area distributions and heights of roughness ridges. The authors of [16, 17] measured (as it is usually done in boundary-layer heat-transfer studies) the mean velocity and temperature profiles for a number of boundary-layer cross sections along the plate, and then they calculated the values of c_f and c_h by an integral method. This method implies differentiation of the experimental data over x, that leads to a considerable loss of accuracy. Perhaps this is exactly the explanation why the experimental values of Nu_x numbers for a smooth plate obtained in [16] and plotted in Fig. 2 are in poor agreement with the values calculated by the theoretical equation for Nu_x derived in [5]. Nevertheless Fig. 2 shows that the agreement between the experimental values of Nu_x and the values calculated with the aid of (13) proves to be more or less satisfactory for both rough plates studied in [16]. As to the data of [17] they may be compared more reliably with the results implied by equation (13). The reason is that in [17] a table is given of the measured values

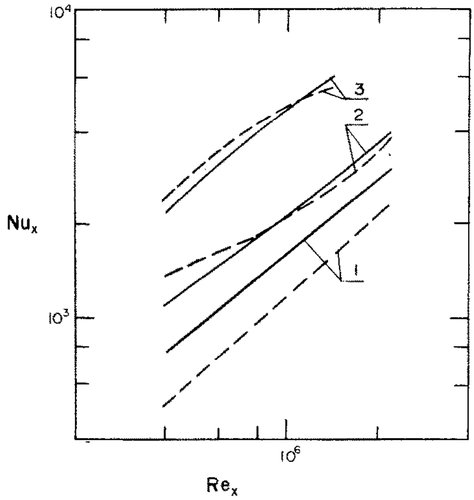

FIG. 2. Nu_x as a function of Re_x according to data of [16] (dotted lines) and according to proposed theoretical equation (solid lines) at $Pr = 0.71$ for smooth (1) and two rough plates: (2) plate with $h = 3$ mm (h_+ ranging from 393 to 570) and $p/h = 4.16$; (3) plate with $h = 1.7$ mm (h_+ ranging from 256 to 303) and $p/h = 7.35$.

of the enthalpy thickness of the boundary layer

$$\Delta_2 = \int_0^{\infty} \frac{U(y)}{U_1}\left[1-\frac{\theta(y)}{\theta_1}\right]dy$$

and of the energy thickness

$$\delta_3 = \int_0^{\infty} \frac{U(y)}{U_1}\left[1-\left(\frac{U(y)}{U_1}\right)^2\right]dy$$

at different x (beginning from $x = 200$ mm). If we calculate the distribution of c_h along the plate by equation (13) and use the measured values of $c_f = c_f(x)$ (given in [17]), we may then determine $\Delta_2(x)$ with the aid of integral relation

$$\Delta_2(x) = \Delta_2(200) + \int_{200}^{x}\left[c_h + \frac{U_1^2}{2c_p\theta_1}\frac{d\delta_3}{dx}\right]dx$$

and compare these values with the measured values of $\Delta_2(x)$. The use of integration instead of differentiation over x makes this comparison more reliable than the one whose results are presented in Fig. 2. The data in Fig. 3 show quite a satisfactory agreement between measured and predicted values of Δ_2 related to the experiments described in [17].

Now we shall compare the results of calculation with the aid of (8) with the data available on heat and mass transfer in pipes and channels with walls covered with two-dimensional roughness. We must take into account, however, that all the experimental data on heat and mass transfer in turbulent pipe flows concern the heat (or mass) transfer coefficient related to the bulk velocity U_b and the bulk temperature (or concentration) θ_b and not to the axial velocity U_1 and the axial temperature (or concentration) θ_1. The ise of U_b instead of U_1 changes nothing in our considerations since the velocity scale U may be chosen arbitrarily in the derivation of equation (4) for c_h (provided the same scale U is used in dimensionless combinations c_h, c_f and Re). Hence we can simply replace U_1 by U_b in all the equations defining c_h, c_f and Re. However the use of θ_1 (i.e. of the difference $\theta_w - \theta_1$ and not of $\theta_w - \theta_b$) is essential to the derivation of (4), and therefore the replacement of $\theta_1 = \theta(L)$ by the bulk value

$$\theta_b = \frac{2}{U_b L^2}\int_0^L (L-y)\theta(y)U(y)\,dy$$

in the definition of the heat- (or mass-) transfer coefficient requires insertion of an additional factor $\Delta^{-1} = (\theta_w - \theta_1)/(\theta_w - \theta_b)$ in the equation for C_h. An approximate estimation of this factor for rough pipes is considered in detail in [6] and we shall not dwell upon it here. The use of the equation for Δ^{-1} derived in [6] and of the above values of α, β, k, b_1'', b_2'' and C' implies the following transformation of (8)

$$C_h = \frac{j_w}{c_p\rho U_b(\theta_w-\theta_b)} = \frac{(c_f/2)^{1/2}}{3.2h_+^{1/4}(Pr^{2/3}+0.3) - 2.12\ln\eta_1 + 4 - \dfrac{3.2}{(1-\eta_1)^2} + 6.7(c_f/2)^{1/2}} \tag{14}$$

where $c_f = 2(u_*/U_b)^2$, $\eta = h/L$. In case of heat transfer into air ($Pr = 0.71$) the latter equation will take the form

$$Nu = C_h Re Pr \approx \frac{Re(c_f/2)^{1/2}}{5h_+^{1/4} - 3\ln\eta_1 + 5.6 - \dfrac{4.5}{(1-\eta_1)^2} + 9.5(c_f/2)^{1/2}}. \tag{15}$$

We must also take into consideration that equations (14) and (15) refer only to pipes with a completely rough wall. However, two-dimensional roughness differs from closely spaced three-dimensional roughness by earlier transition to a completely rough flow. It is known that the wall covered with sand or similar three-dimensional roughness can be considered dynamically completely rough only at $h_+ = hu_*/\nu$ exceeding the "threshold" value $h_+^{(0)}$ where $h_+^{(0)} \approx 100$. At the same time the friction data of [10, 14] show that for two-dimensional roughness the skin-friction coefficient c_f assumes a constant value (i.e. the wall is completely rough) even when h_+ is between 25 and 35. As to the heat and mass transfer, transition to "completely rough flow" described by equation (14) proceeds apparently still earlier (i.e. at lower values of h_+). This fact is clearly seen in the graphs representing Dawson and Trass's data [14] on mass transfer at a rough wall for very large Pr (Figs. 10–16): most of them show a sharp change of the slope in the Nu vs Re curve at relatively low values of h_+ (most frequently close to 10) and above this bend point equation (14) turns to be applicable with quite a satisfactory accu-

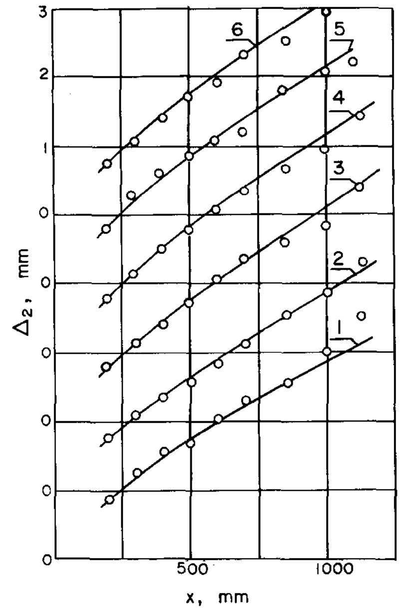

FIG. 3. Δ_2 vs x for six rough plates according to data of [17] (points) and according to calculations (solid lines). $Pr = 0.71$; six zeros on the ordinate axis show the origin of values for six curves.

No.	h (mm)	Range of h_+	p/h
1	3	307–540	2.65
2	3	363–583	3.30
3	1.9	263–320	4.21
4	2.4	318–465	4.13
5	4	538–840	4.18
6	4	563–825	4.18

racy. Since the main variation of the temperature (or concentration) takes place at large Pr in a very thin wall-adjacent layer, whose thickness decreases with increase of Pr, it may be supposed that the equation for $\beta(Pr, h_+)$ describing a "completely rough flow" may be generally applied beginning from the "threshold value" $h_+ = h_+^{(0)}$, where $h_+^{(0)}$ depends on Pr and decreases with increasing Pr. At present, however, there are no data that would allow a detailed study of the problem on the boundary between a "completely rough flow conditions" and "transitional flow conditions" and on the heat- (or mass-) transfer properties of such "transitional flows". Therefore we shall simply take into account that if the true roughness height h (that does not consider the possibility of a great difference in shapes of roughness elements) is replaced by the height h_s of equivalent (i.e. causing the same friction at rather large Re sand roughness), then the bend in the Dawson and Trass's curves $Nu = Nu(Re)$ in all cases takes place at $h_{s+} = h_s u_* / \nu$ close to 25. Since for all other experimental data analysed in this paper the value of $h_{s+}^{(0)} = 25$ also turns out to be a reasonable estimate of the "threshold value" of h_+ controlling the transition to "completely rough flow conditions", equations (14) and (15) will be applied only at $h_{s+} \geqslant 25$. The Reynolds number corresponding to $h_{s+} = 25$ is shown with a vertical dotted line in all of the following figures. For the measurements at $h_{s+} < 25$ the simplest method of linear interpolation is used which is based on the replacement of $\beta(Pr, h_+)$ in equation (4) for c_h by the following interpolation value

$$\beta = \frac{h_{s+}}{25}\beta_r + \left(1 - \frac{h_{s+}}{25}\right)\beta_s. \tag{16}$$

Here β_s is given by equation (5) while β_r is given by (7) with $k = 1/4$, $b_1'' = 3.2$, $b_2'' = 0.3$ and $C' = 3.5$ (cf. a similar reasoning in [6] where it is, however, assumed that $h_{s+}^{(0)} = 100$). Transition from c_h to C_h upon determination of β and in the case of transitional flow requires only insertion of an additional factor Δ^{-1} (whose value is given in [6]) into the equation for a heat- and mass-transfer coefficient.

Figure 4 shows that equation (15) describes quite satisfactorily (and much better than the equation for C_h suggested in [6]) Nunner's experimental data [9] for pipes with walls roughened by removable circumferential rings of different shapes. Let us note, however, that in the case of heat transfer into the air satisfactory agreement with experimental data can also be achieved based on some theoretical models quite different from the one used in the present work (cf. e.g. the models

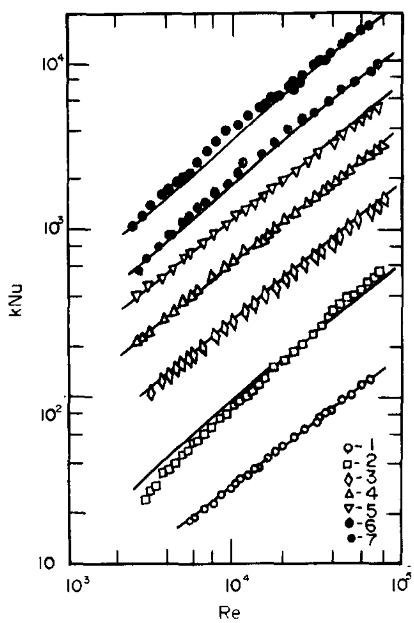

FIG. 4. kNu (where factor k is included in order to avoid overlapping of data points) vs Re according to data of [9] at $Pr = 0.71$. Solid lines for rough pipes represent calculations with the aid of equation (15)

No.	η_1	Range of h_+	Range of h_{s+}	p/h	k
1	0	0	0	—	1
2	0.182	26.4–701	18.3–486	13.5	2
3	0.167	44.8–1157	189–4883	6.2	4
4	0.164	42.0–1210	239–6900	5.7	8
5	0.162	34.0–1046	137–4213	7.0	16
6	0.0805	14.9–402	69.9–1890	6.2	32
7	0.0805	12.2–316	38.6–1003	7.3	64

of Galin [18] and Migai [19] who supposed that the structure of the viscous sublayer in the gaps between the ridges of closely spaced two-dimensional roughness does not differ considerably from that on a smooth wall). Nevertheless, it seems that in the case of heat transfer into the air the above equation (15) also leads to somewhat better agreement with experimental data than all of the equations for C_h suggested in the literature we have seen. However, only the comparison of the above equation with experimental data covering a wide range of Pr numbers and a great number of roughnesses of different types may provide a decisive verification of this equation.

In this respect the references [20–22] are of considerable interest. These works describe the results of a study of heat transfer in pipes with two-dimensional roughness of a special kind, namely, with transverse annular protrusions made by pressing the pipe wall from the outside with a special roll. Engineering advantages of utilizing such a roughness to enhance heat transfer are discussed in detail in book [1]. The results of [20–22] refer to a series of Prandtl numbers, since the data include the heat-transfer coefficients for air [20], water [21], and pure water and aqueous glycerine solution [22] flows. The comparison of the measured Nu numbers with the values calculated by (14) [or with similar equation for transitional flows based on relation (16)] is shown in Figs. 5–7. In calculation of the Nu numbers in Figs. 6 and 7, temperature dependent Prandtl numbers for water and glycerine solutions were taken for the mean tempera-

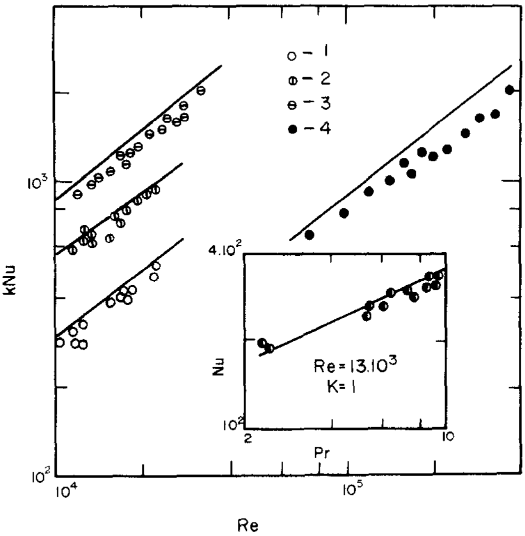

FIG. 6. kNu vs Re, and Nu vs Pr at $Re = 1.3 \cdot 10^4$ (insertion at the lower right corner) according to data of [21]. Solid lines are calculated with the aid of proposed theoretical equation.

No.	η_1	Range of h_+	Range of h_{s+}	p/h	Pr	k
1	0·095	81.2–172	512–1083	10.6	9.8	1
2	0.095	82.7–158	426–816	15.6	8.7 & 2.3–9.1	2 & 1
3	0.095	80.3–216	351–943	20.5	6.5	4
4	0.050	216–1034	916–4377	20.5	3.1	1

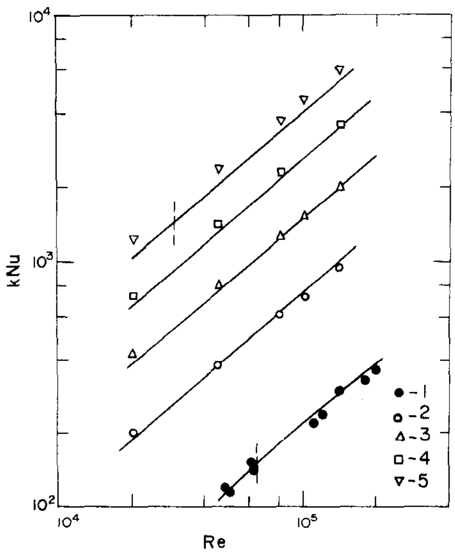

FIG. 5. kNu vs Re according to data of [20]. $Pr = 0.71$; solid lines correspond to equation (15).

No.	η_1	Range of h_+	Range of h_{s+}	p/h	k
1	0.119	181–753	18.0–75.0	2.5	1
2	0.04	46.9–328	270–1886	8.4	2
3	0.04	47.5–333	286–2002	12.0	4
4	0.04	38.7–271	107–748	16.7	8
5	0.04	29.7–208	16.5–116	33.3	16

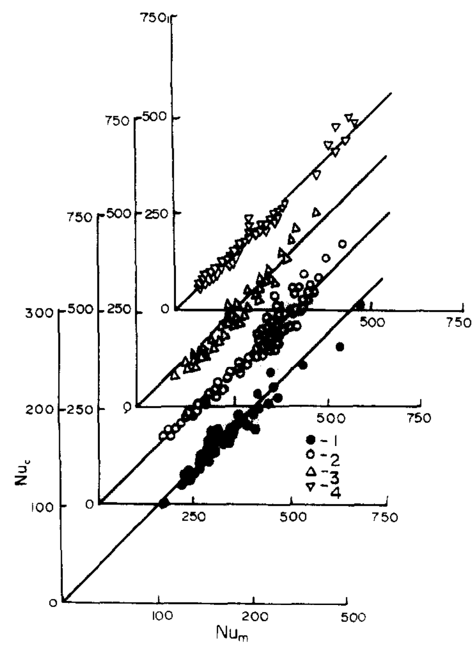

FIG. 7. Comparison of measured values Nu_m with calculated values Nu_c for the data of [22].

No.	η_1	Range of h_+	Range of h_{s+}	p/h	Pr
1	0	0	0	—	10–43
2	0.078	42.6–339	106–842	30	3–28
3	0.054	12.1–123	4.5–46.0	37	4–21
4	0.034	7.6–91.5	5.2–79.7	13	2–32

ture $\theta_0 = (\theta_w + \theta_b)/2$. To plot together all the data of Kalinin and Yarkho [22] for a wide range of Pr numbers in Fig. 7, the "comparison chart" was used with measured Nu_m values on the abscissa and calculated Nu_c numbers on the ordinate. In Figs. 6 and 7 the scatter of points indicates somewhat worse agreement between measured and calculated Nu numbers than in the case of heat transfer through air. This fact may, in particular, be attributed to disregard of non-uniform temperature distribution over roughness ridges which is apparently of importance for water and aqueous solution flows along a rough wall. Nevertheless even in cases shown in Figs. 6 and 7 the agreement between the measurements and calculations is rather satisfactory from an engineering point of view.

The related two-dimensional "repeated-rib" roughness consisting of annular protrusions of rectangular profile was used by Webb, Eckert and Goldstein [10] who studied heat transfer between pipe walls and turbulent flow of air, water or butyl alcohol (at $0.7 \leqslant Pr \leqslant 37$). Figure 8 shows that in this case, too, experimental data agree satisfactorily with calculations based on equations suggested in the present paper.

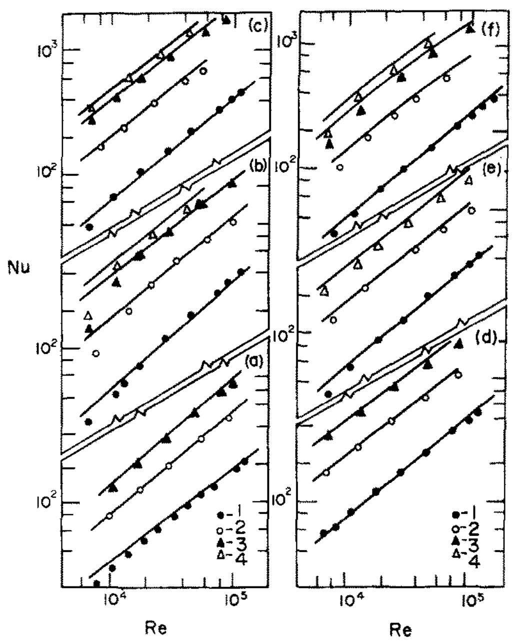

FIG. 8. kNu vs Re according to data of [10]. Solid lines are calculated with the aid of proposed theoretical equation.

		1	2	3	4
	Pr	0.71	4.9	20.9	35
(a)	h_+	0	0	0	0
$\eta_1 = 0$	h_{s+}	0	0	0	0
(b)	h_+	7.0–119	7.8–97.6	11.1–52.8	6.6–42.5
$\eta_1 = 0.02$	h_{s+}	43.9–748	49.0–616	69.8–333	41.4–268
$p/h = 10$					
(c)	h_+	17.6–303	22.0–150	18.4–234	18.2–110
$\eta_1 = 0.04$	h_{s+}	154–2641	191–1308	160–2040	158–955
$p/h = 10$					
(d)	h_+	47.7–858	48.9–587	48.9–587	—
$\eta_1 = 0.08$	h_{s+}	429–7709	439–5271	439–5271	—
$p/h = 10$					
(e)	h_+	15.9–279	17.8–234	15.0–218	—
$\eta_1 = 0.04$	h_{s+}	83.9–1476	94.1–1238	79.6–1153	—
$p/h = 20$					
(f)	h_+	13.9–268	14.5–108	11.8–163	11.6–77.0
$\eta_1 = 0.04$	h_{s+}	28.0–542	29.3–219	23.9–329	23.5–156
$p/h = 40$					

It has already been noted that high Prandtl numbers in Dawson and Trass's studies [14] are of special interest. These authors have electrochemically measured mass transfer at a rough (and, to compare with, at a smooth) upper wall of a rectangular channel at very high Pr numbers ($390 \leqslant Pr \leqslant 4585$). A series of rough surfaces with geometrically similar two-dimensional protrusions of different height have been used in this study and in addition to the mass-transfer coefficient c_h the skin friction coefficient c_f has also been measured. When analysing Dawson and Trass's data it should be remembered that, strictly speaking, equation (14) derived for a circular pipe flow is not applicable to the flow in a rectangular channel with only one rough wall. However, at very large Prandtl numbers the mean concentration $\theta(y)$ changes sharply in a very thin layer adjacent to the wall, and then remains almost constant. Therefore θ_b in Dawson and Trass's experiments does not practically differ from the maximum concentration θ_1 at the channel center (though both θ_b and θ_1 differ greatly from θ_w). Hence, the correcting factor $\Delta^{-1} = (\theta_w - \theta_1)/(\theta_w - \theta_b)$ is very close to unity and its deviation from the value of Δ^{-1} (that is also very close to unity) in case of a circular pipe flow at the same Pr is of no real significance. In other words, inaccuracy of equation (14) in case of a rectangular channel flow due to the use of the factor Δ^{-1} calculated for a circular pipe appears to be quite negligible at $Pr \geqslant 390$. A crude estimate of a possible order of the magnitude of the corresponding error shows that this error lies far beyond the limits of accuracy of the calculation method recommended in the present paper. Therefore in Figs. 9–16 the data of [14] are compared with the calculations by (14) and (16). This comparison shows that agreement between the measured and calculated values, both for a smooth wall and for all completely rough walls, proves to be quite satisfactory according to Dawson and Trass's data. Let us note that a good agreement in a smooth-wall case is of special interest since it demonstrates most clearly the correctness of the exponent at Pr in the equation for β. As far as transitional flows along a slightly rough wall are considered, the agreement seems to be more poor in a number of cases, which apparently indicates insufficient accuracy of a fixed "threshold value" $h_{s+}^{(0)} = 25$ and of the simplest linear interpolation between smooth and completely rough wall flows.

Figures 2–16 show that the method of calculations suggested in the present paper allows prediction, with a satisfactory accuracy, of the values of heat- and mass-transfer coefficients for a great variety of turbulent flows along the walls covered with different two-dimensional roughness. In particular, the method proves to be applicable within a wide range of p/h values (from $p/h \approx 4$ to $p/h \approx 40$), Re (from $3 \cdot 10^3$ to $2 \cdot 10^5$), Pr (from 0.7 to 4585) and h_+ (from 10 to 4000). It should also be emphasized that the results of the paper imply that the effect of roughness plays the

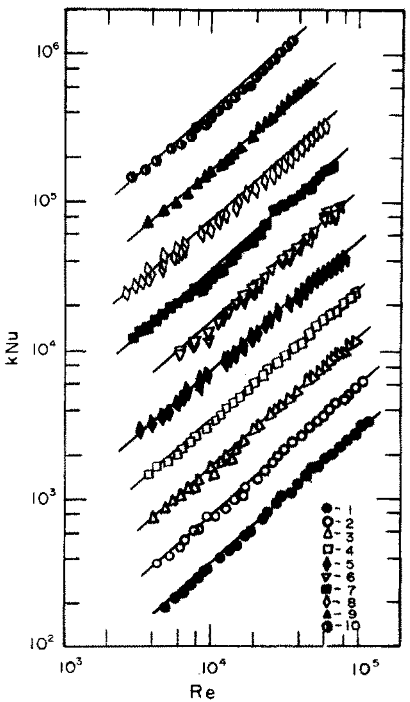

FIG. 9. kNu vs Re for mass transfer at a smooth wall according to data of [14]. Solid lines are calculated with the aid of equation suggested in [5].

No.	1	2	3	4	5	6	7	8	9	10
Pr	393	464	556	670	814	1004	1246	1847	2858	4585
k	1	2	4	8	16	32	64	128	256	512

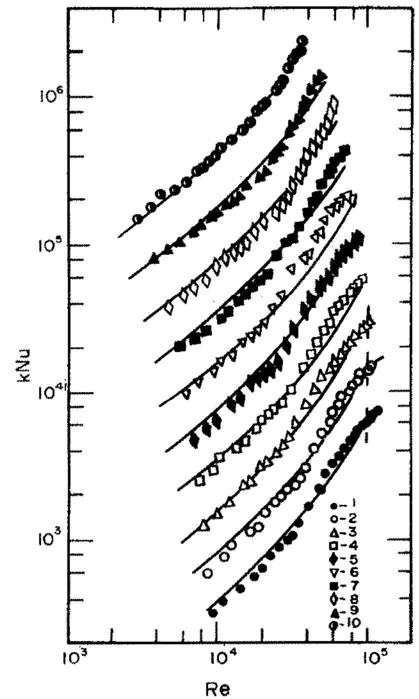

FIG. 10. kNu vs Re according to data of [14] for a plate $R1$ ($\eta_1 = 0.0044$ where $\eta_1 = h/L$, and L is equivalent radius of channel, i.e. radius of a circular pipe with the same cross section; $p/h = 4.5$). Solid lines are calculated with the aid of proposed theoretical equation.

No.	Pr	Range of h_+	Range of h_{s+}	k
1	393	15.5–1.2	28.5–2.3	1
2	464	14.4–1.1	26.4–2.1	2
3	556	13.3–1.1	24.3–1.9	4
4	670	12.2–1.0	22.3–1.8	8
5	814	11.1–0.9	20.4–1.6	16
6	1004	10.1–0.8	18.5–1.5	32
7	1246	9.1–0.7	16.8–1.3	64
8	1847	7.5–0.6	13.8–1.1	128
9	2858	6.0–0.5	11.1–0.9	256
10	4585	4.8–0.4	8.8–0.7	512

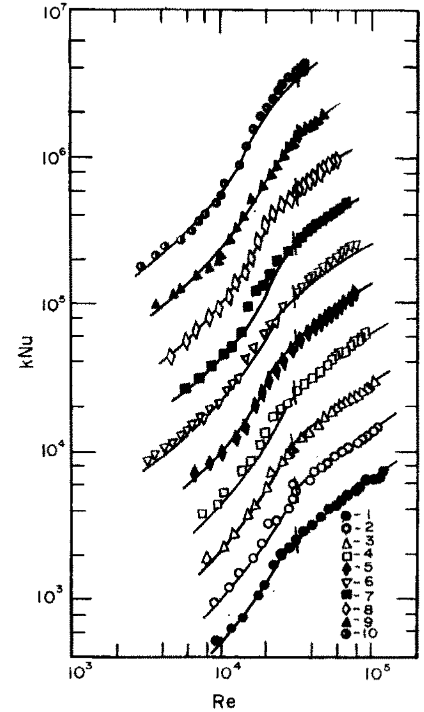

FIG. 11. The same as in Fig. 10 but according to data for plate $R2$ ($\eta_1 = 0.0104$, $p/h = 3.75$).

No.	Pr	Range of h_+	Range of h_{s+}	k
1	393	43.5–3.4	97.7–7.7	1
2	464	40.3–3.2	90.6–7.1	2
3	556	37.1–2.9	83.4–6.6	4
4	670	34.1–2.7	76.6–6.0	8
5	814	31.2–2.5	70.1–5.5	16
6	1004	28.3–1.1	63.6–2.5	32
7	1246	25.6–2.0	57.5–4.5	64
8	1847	21.0–1.7	47.3–3.7	128
9	2858	16.9–1.3	38.0–3.0	256
10	4585	12.7–1.0	28.6–2.4	512

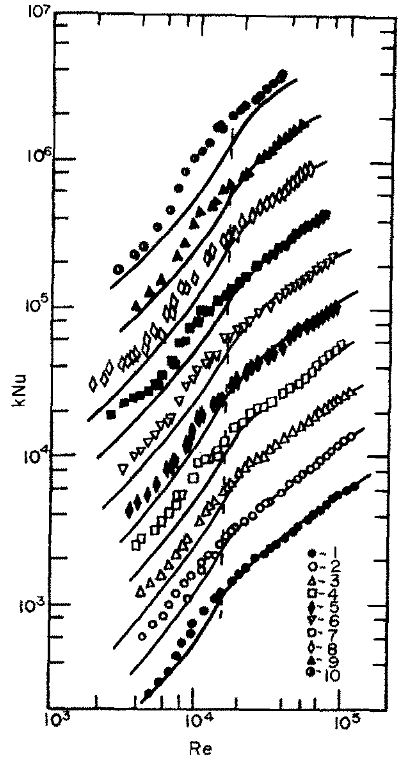

FIG. 12. The same as in Fig. 10, but according to data for plate $R3$ ($\eta_1 = 0.016$; $p/h = 3.75$).

No.	Pr	Range of h_+	Range of h_{s+}	k
1	393	70.9–2.9	144–5.9	1
2	464	65.8–2.7	134–5.4	2
3	556	60.5–2.4	123–5.0	4
4	670	55.6–2.2	113–4.5	8
5	814	50.9–2.0	103–4.1	16
6	1004	46.1–1.8	93.7–3.7	32
7	1246	41.8–1.6	84.8–3.3	64
8	1847	34.3–1.2	69.7–2.3	128
9	2858	27.6–2.2	56.1–4.4	256
10	4585	21.8–1.7	44.2–3.5	512

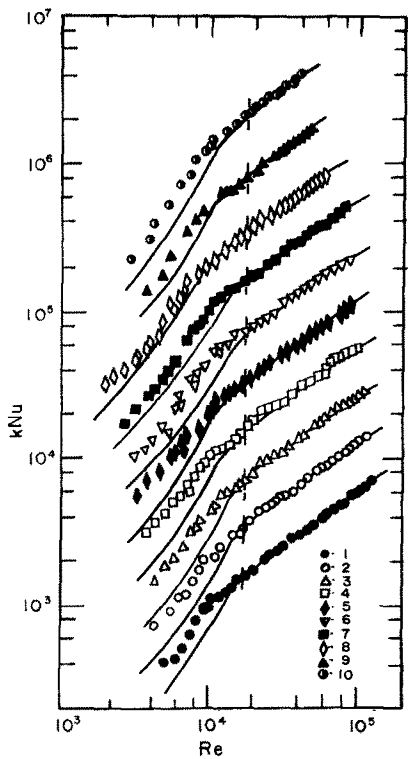

FIG. 13. The same as in Fig. 10, but according to data for plate $R4$ ($\eta_1 = 0.22$; $p/h = 3.64$).

No.	Pr	Range of h_+	Range of h_{s+}	k
1	393	101–4.1	182–7.4	1
2	464	93.9–3.8	169–6.9	2
3	556	86.4–3.5	156–6.3	4
4	670	79.4–3.2	143–5.7	8
5	814	72.6–2.9	131–5.2	16
6	1004	65.9–2.6	119–4.7	32
7	1246	59.6–2.3	107.5–4.2	64
8	1847	49.0–1.6	88.3–3.0	128
9	2858	39.4–3.1	71.0–5.6	256
10	4585	31.1–2.5	56.1–4.4	512

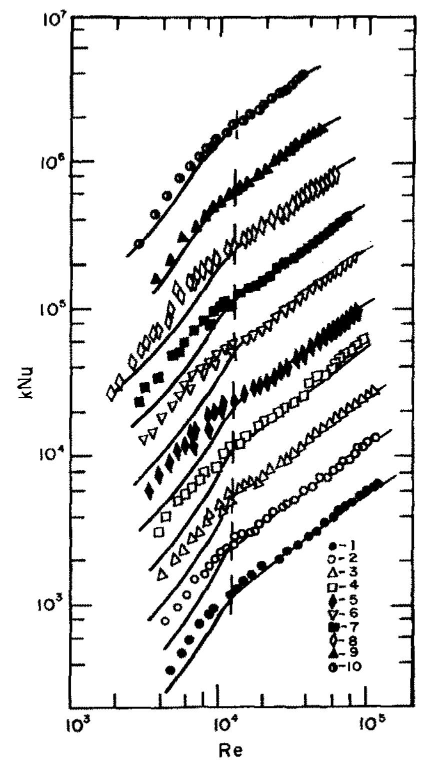

FIG. 14. The same as in Fig. 10, but according to data for plate $R5$ ($\eta_1 = 0.0272$; $p/h = 3.68$).

No.	Pr	Range of h_+	Range of h_{s+}	k
1	393	130–5.3	233–9.5	1
2	464	121–4.9	216–8.7	2
3	556	111–4.5	199–8.0	4
4	670	102–4.1	182–7.3	8
5	814	93.4–3.7	167–6.6	16
6	1004	84.7–3.3	151–6.0	32
7	1246	76.7–3.0	137–5.4	64
8	1847	63.0–2.1	113–3.8	128
9	2858	50.7–4.0	90.6–7.1	256
10	4585	40.0–3.2	71.5–5.6	512

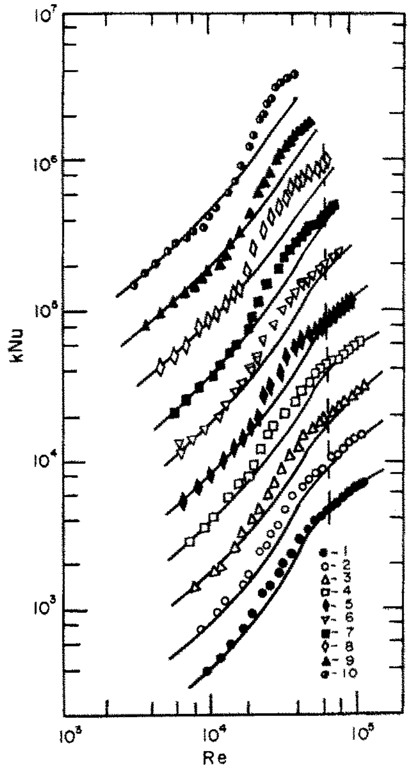

FIG. 15. The same as in Fig. 10, but according to data for plate $R6$ ($\eta_1 = 0.008$; $p/h = 3.75$).

No.	Pr	Range of h_+	Range of h_{s+}	k
1	393	30.1–2.2	46.9–3.4	1
2	464	27.9–2.0	43.5–3.2	2
3	556	25.7–1.9	40.0–2.9	4
4	670	23.6–1.7	36.8–2.7	8
5	814	21.6–1.6	33.7–2.4	16
6	1004	19.6–1.6	30.5–2.4	32
7	1246	17.7–1.4	27.6–2.2	64
8	1847	14.6–1.2	22.7–1.8	128
9	2858	11.7–0.9	18.3–1.4	256
10	4585	9.3–0.7	14.4–1.1	512

double role in heat and mass transfer. On the one hand, disturbances produced by roughness ridges intensify heat and mass transfer; on the other, flow deceleration in the gaps between ridges deteriorates heat and mass transfer. The second effect is especially pronounced at large Pr and Re values and it can even make the rate of heat and mass transfer from a rough wall lower than that from a smooth wall at the same Pr and Re numbers. It is clear that at very large Pr and Re numbers the first term in the denominator of the RHS of (15) is dominating, and thus $(c_h)_r \sim Re^{-1/4}Pr^{-2/3}$ (because $c_f = \text{const}$ and $h_+ \sim Re$), while for a smooth wall the first term of equation (5) for β is dominating and therefore $(c_h)_s \sim (c_f)^{1/2}Pr^{-2/3}$. Hence it follows that $(c_h)_s \sim Re^{-1/8}Pr^{-2/3}$, $(c_h)_r/(c_h)_s \sim Re^{-1/8}$ if the known Blasius friction law is valid. The latter result is close to the experimental result of Dawson and Trass [14] [according to these authors $(c_h)_r/(c_h)_s \sim Re^{-0.1}$ at large Pr and large enough Re].

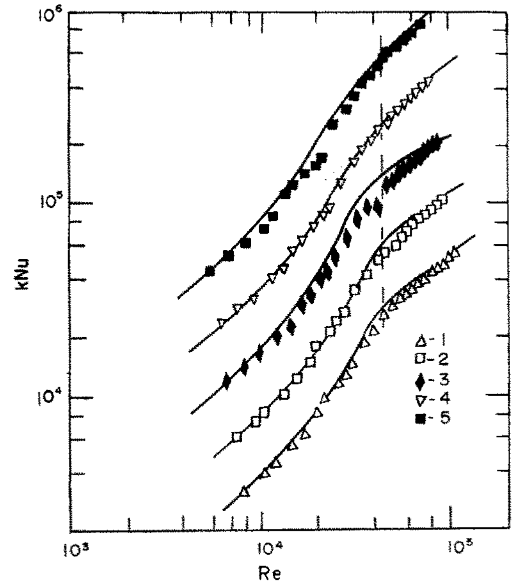

FIG. 16. The same as in Fig. 10, but according to data for plate $R8$ ($\eta_1 = 0.008$; $p/h = 7.5$).

No.	Pr	Range of h_+	Range of h_{s+}	k
1	556	27.1–2.1	58.0–4.6	8
2	670	24.9–2.0	53.3–4.2	16
3	814	22.8–1.8	48.7–3.8	32
4	1004	20.6–1.6	44.2–3.5	64
5	1246	18.7–1.5	40.0–3.2	128

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TRANSFERT TURBULENT DE MASSE ET DE CHALEUR SUR UNE PAROI RUGUEUSE A SILLONS PARALLELES

Résumé—On sait que l'équation générale du coefficient de transfert thermique et massique entre une paroi rugueuse et un fluide en écoulement turbulent peut être obtenue à l'aide de considérations générales de dimension et de similitude auxquelles s'ajoutent quelques arguments physiques. L'équation est donnée ici dans le cas d'une paroi recouverte d'une rugosité bidimensionnelle faite d'arêtes parallèles, largement espacées et perpendiculaires à la direction de l'écoulement. Les coefficients constants de l'équation sont estimés à partir des données disponibles sur les profils moyens de température ou de concentration, pour les écoulements turbulents sur paroi à rugosité bidimensionnelle. Les résultats du calcul s'accordent avec toutes les expériences sur le transfert turbulent de chaleur et de masse dans les tuyaux et les canaux avec des sillons parallèles et régulièrement espacés, ainsi que pour les plaques ayant ce type de rugosité.

TURBULENTER WÄRME- UND STOFFÜBERGANG AN EINER WAND MIT PARALLELEN RAUHIGKEITSERHEBUNGEN

Zusammenfassung—Bekanntlich kann die allgemeine Gleichung für den Wärme (Stoff)-Übergang zwischen einer rauhen Wand und einer turbulenten Fluidströmung mit Hilfe von Dimensions- und Aehnlichkeitsbetrachtungen unter Zuhilfenahme einiger zusätzlicher physikalischer Parameter hergeleitet werden. Für den speziellen Fall einer Wand, die mit zweidimensionalen Rauhigkeiten in der Form paralleler, weit voneinander entfernter Erhebungen, welche im rechten Winkel zur Strömungsrichtung verlaufen, wird diese Gleichung hier abgeleitet. Aus den vorhandenen Daten der mittleren Temperatur—oder Konzentrationsprofile in wandturbulenten Strömungen über zweidimensionale Rauhigkeiten werden die Konstanten der Gleichung angenähert ermittelt. Die Rechenergebnisse stimmen befriedigend überein mit allen experimentellen Daten des turbulenten Wärme- und Stoffübergangs in Rohren und Kanälen mit regelmässig, sich wiederholenden parallelen Rauhigkeiten, sowie an Platten mit zweidimensionalen Rauhigkeiten derselben Form.

ТУРБУЛЕНТНЫЙ ТЕПЛО- И МАССОПЕРЕНОС ОТ СТЕНКИ, ПОКРЫТОЙ ПАРАЛЛЕЛЬНЫМИ ГРЕБНЯМИ ШЕРОХОВАТОСТИ

Аннотация — Общая формула для коэффициента тепло- или массопереноса от шероховатой стенки к турбулентному течению жидкости, вытекающая из анализа размерностей и некоторых дополнительных соображений физического характера, конкретизируется в применении к случаю стенки, покрытой двумерной шероховатостью в виде сравнительно редких параллельных гребней, перпендикулярных направлению средней скорости. Значения постоянных коэффициентов, входящих в предлагаемую формулу, приближенно оцениваются на основе имеющихся данных о профилях средней температуры или концентрации в пристенных турбулентных течениях над стенкой с двумерной шероховатостью. Полученные результаты позволяют с удовлетворительной точностью описать многочисленные измерения турбулентного тепло- и массопереноса в трубах и каналах со стенками, покрытыми гребнями шероховатости, и на пластинках с такого же типа шероховатостью, обтекаемых турбулентным потоком.